

Figure 1. Perspective and cross-section of the GaAs/InGaAs photonic-crystal slab (period  $a$ , hole radius  $r$ , slab thickness  $t$ ) with the bound-state-in-continuum (BiC) mode field profile sketched in blue concentric contours.

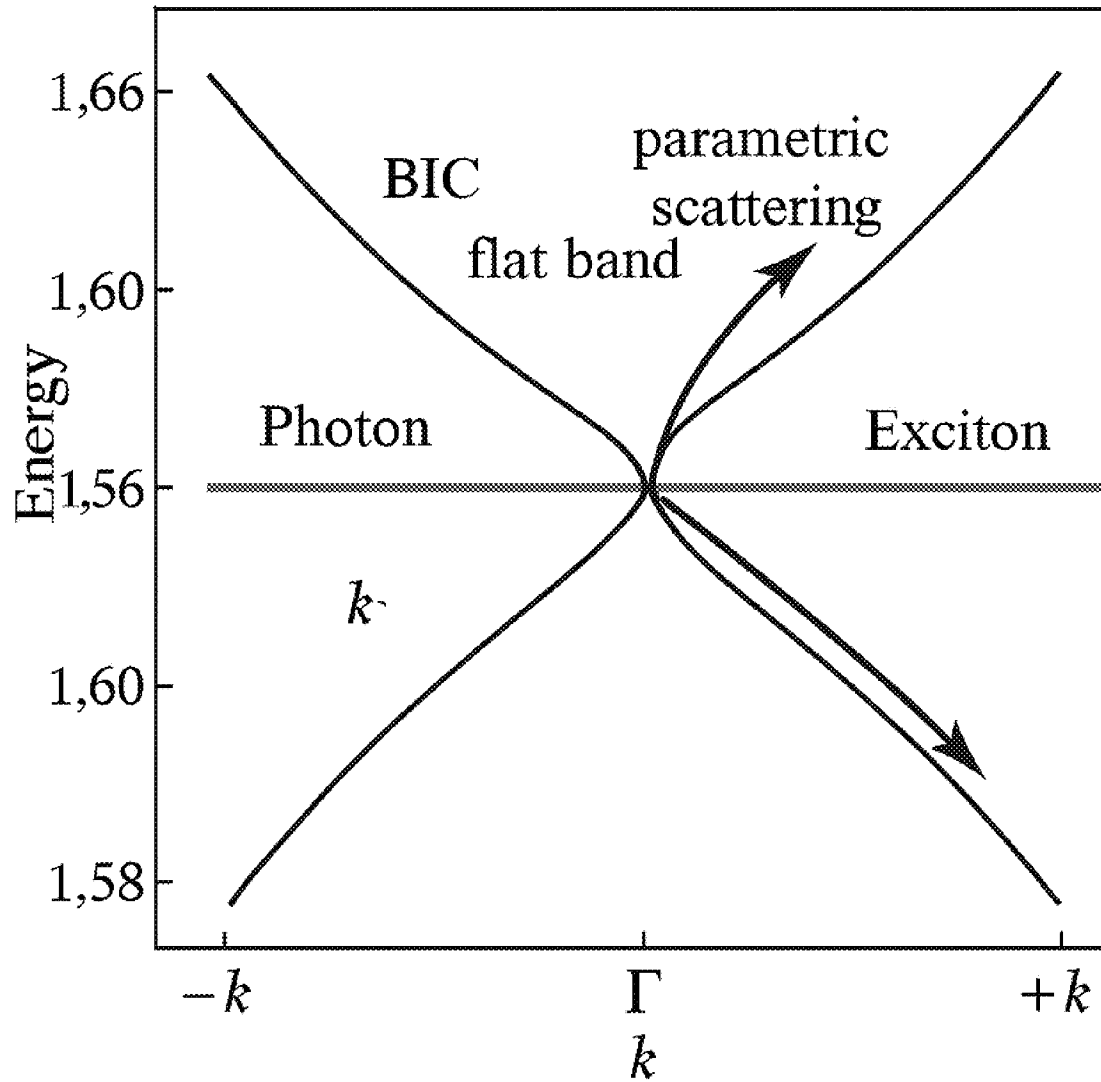


Figure 2. Calculated polariton dispersion at the  $\Gamma$ -point: the symmetry-protected BiC appears as an almost flat blue band crossing the exciton resonance (horizontal), while the photon-like branches (black) curve upward and downward. Arrows mark parametric scattering from the  $k \approx 0$  condensate into  $\pm k$  side-modes that lock to form the supersolid lattice period  $\Lambda = \pi/k$ .

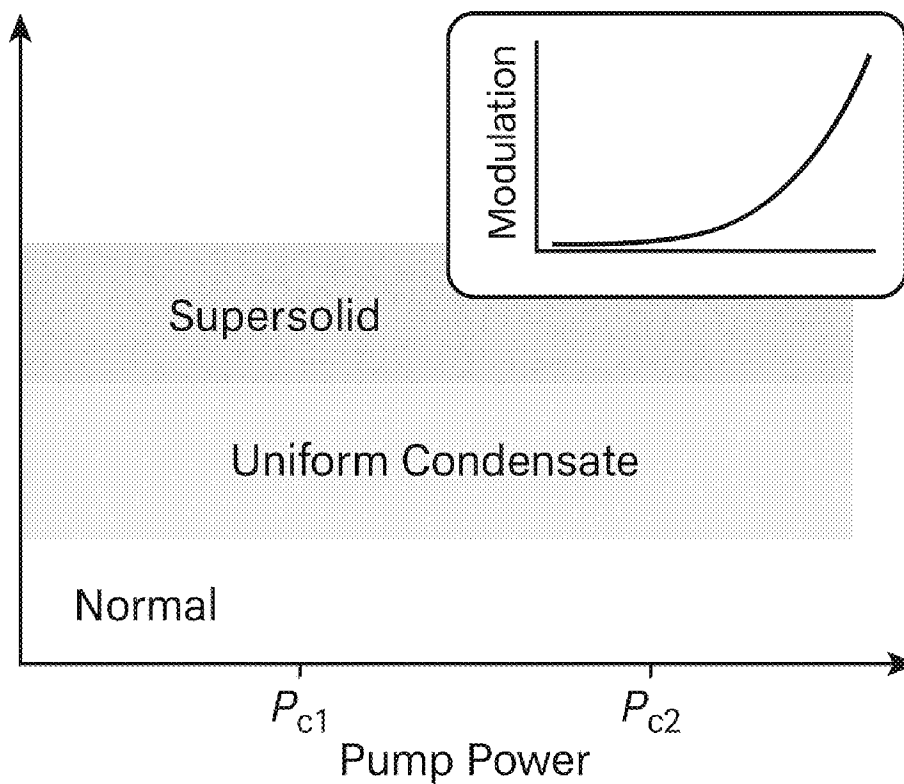
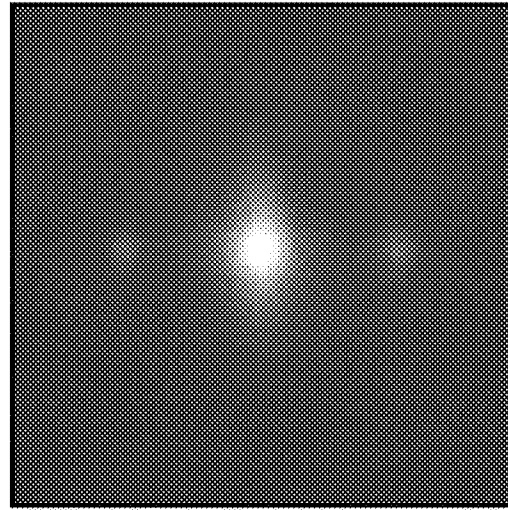
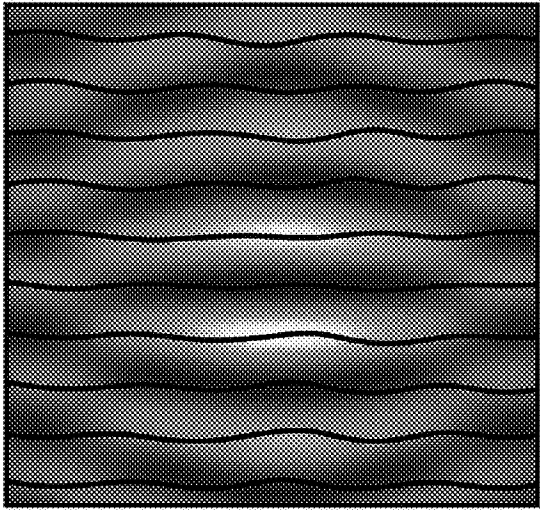


Figure 3. Pump-power phase diagram:

- Below  $P_{c1}$  – no condensate (normal).
- $P_{c1} \leq P < P_{c2}$  – uniform polariton condensate.
- $P \geq P_{c2}$  –  $\chi^3$  self-locking drives a supersolid with density modulation depth that grows with pump power (inset).

Phase



Emission

Fourier transform

Phase coherence and density modulation

Figure 4. Left: Real-space emission from the supersolid polariton condensate, showing periodic intensity fringes (colour map) overlaid with equal-phase contours (black), evidencing long-range phase coherence.

Right: Fourier transform of the same image reveals the central condensate peak at  $k \approx 0$  plus symmetric side-peaks at  $\pm k$ , confirming the self-organised density lattice.

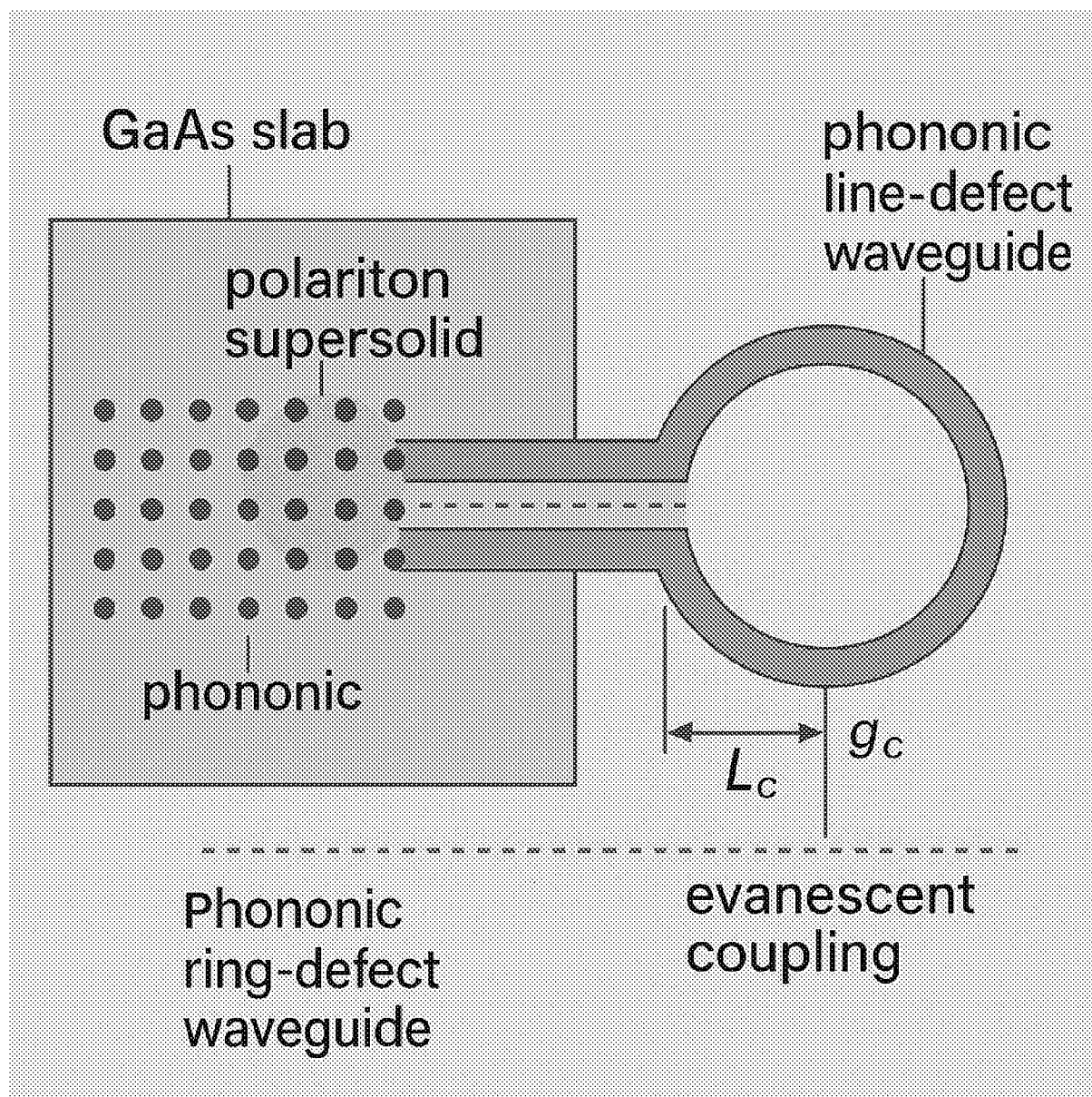


Figure 5. Plan-view layout: the photonic-crystal GaAs slab hosts the polariton supersolid (yellow region). A neighbouring one-row line-defect forms a suspended phononic waveguide that couples (gap  $g_c$ , length  $L_c$ ) to a circular phononic ring resonator, enabling extraction and routing of the supersolid's Goldstone-mode vibrations.

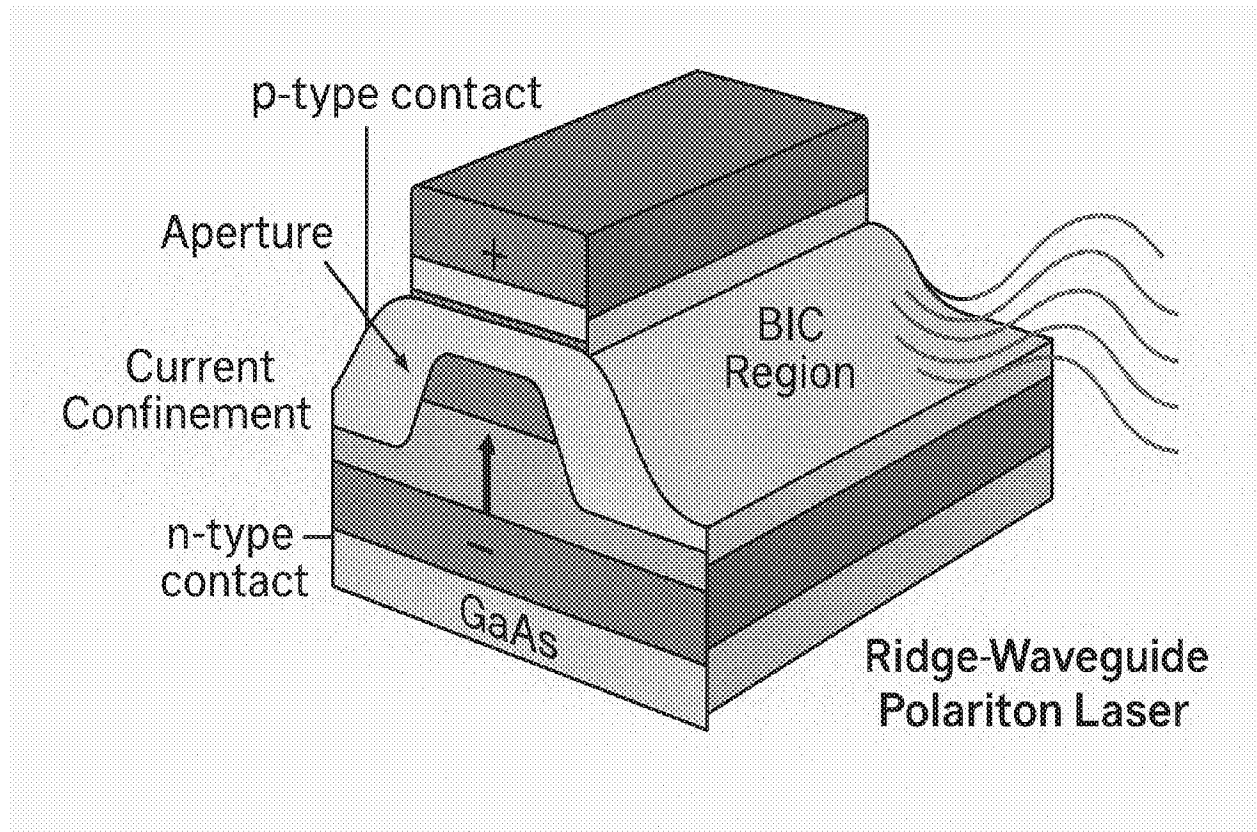


Figure 6. Electrical-injection ridge-waveguide polariton laser variant. Current is confined through an aperture into the BIC region, inducing a supersolid condensate under drive current  $I \geq I_{c2}$ . Contacts and confinement layers are labelled.

## Heterogeneous stack cross-section (III-V / SiN / LiNbO<sub>3</sub>)

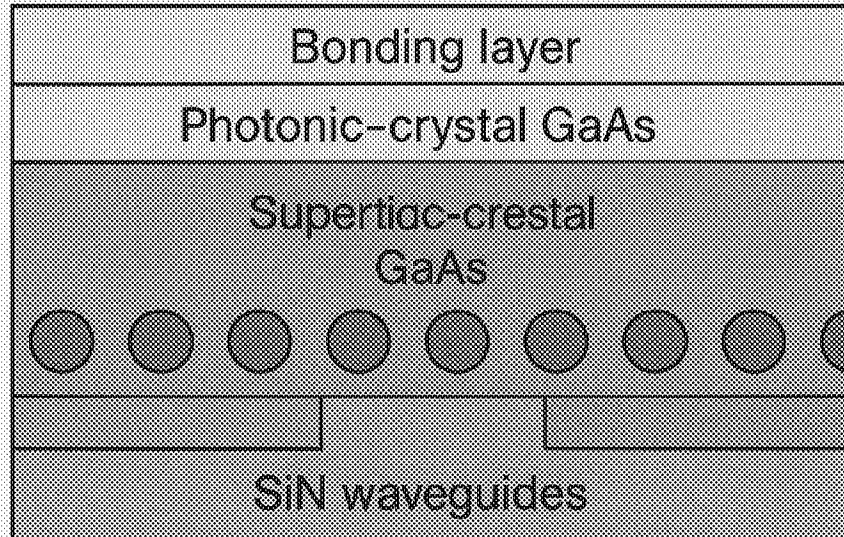


Figure 7. Cross-section of a heterogeneous III-V / SiN / LiNbO<sub>3</sub> stack: a bonded thin-film LiNbO<sub>3</sub> layer (top) overlies the photonic-crystal GaAs slab that hosts the supersolid polariton lattice (etched holes). Underneath, SiN waveguides route on-chip signals; the bonding layer provides optical and acoustic coupling between tiers.

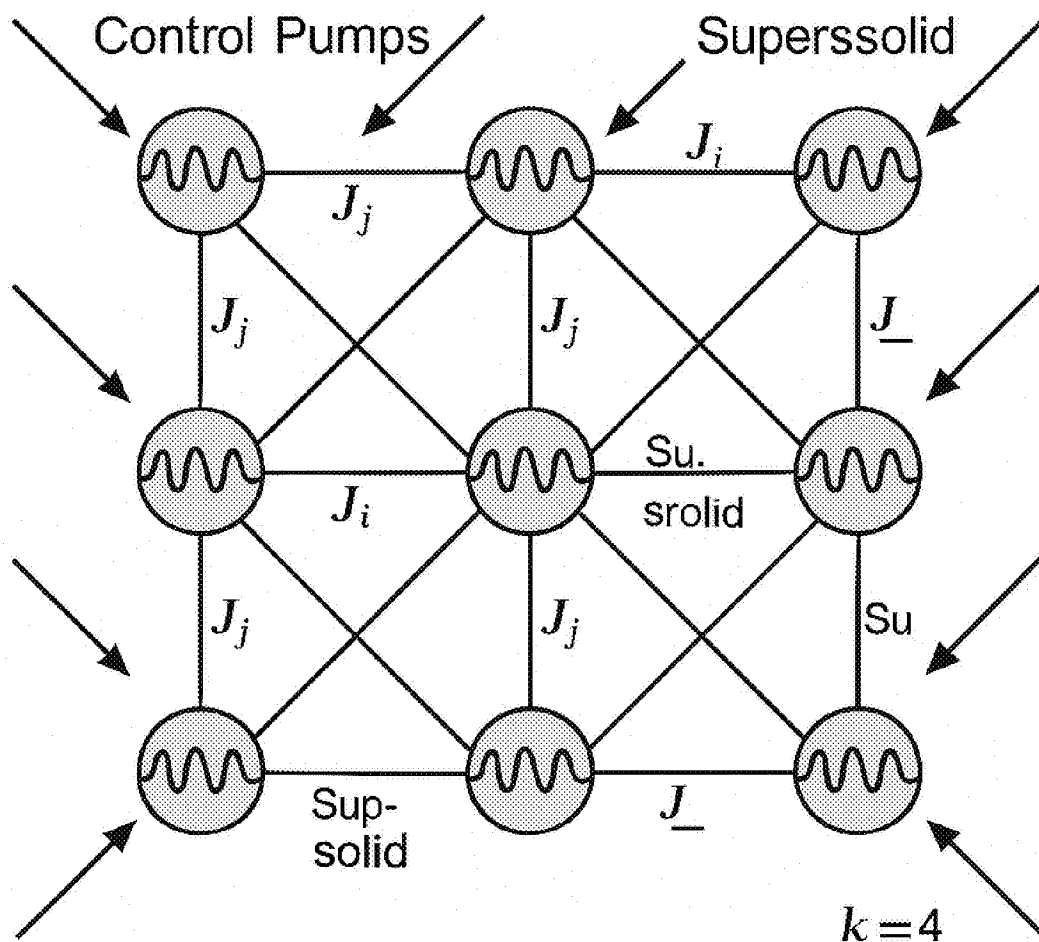
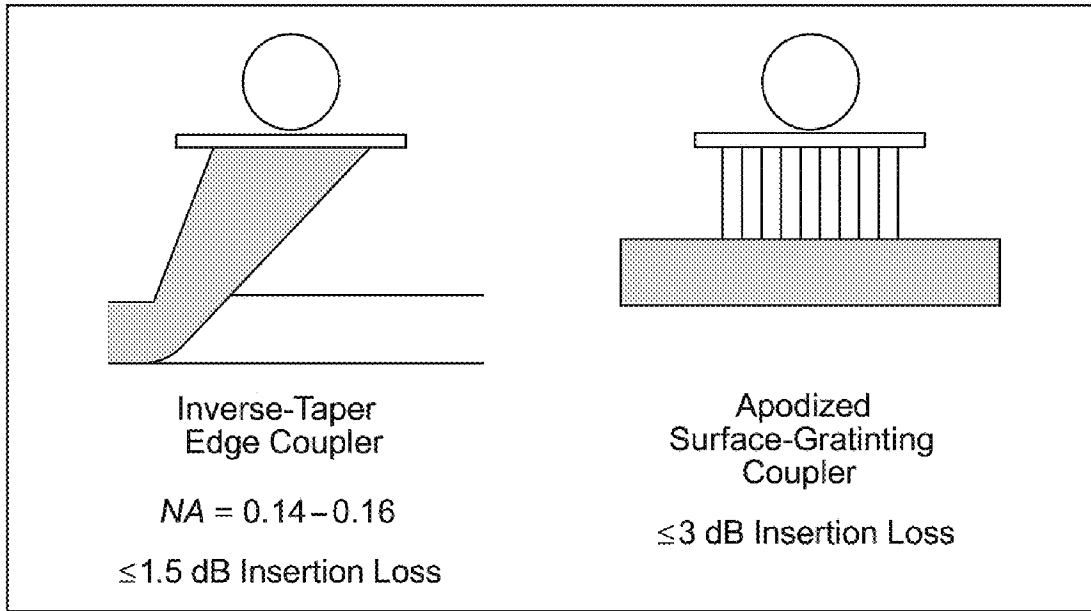


FIG. 8 Array of supersolid cells with couplings and control pumps

Figure 8. Array architecture for quantum simulation: a  $4 \times 4$  grid of polariton supersolid cells (yellow) with programmable nearest-neighbour couplings  $J_{ij}$ . Diagonal arrows mark independent pump spots that tune each cell's onsite potential and inter-cell coupling strength, enabling implementation of Ising or Bose-Hubbard Hamiltonians.



**FIG. 9**

**Edge-Taper and Surface-Grating Fibre-Coupler Geometries**

Figure 9. Edge-taper (left) and apodised surface-grating (right) fibre-coupler geometries with key specifications:  $NA \approx 0.14-0.16$  and  $\leq 1.5$  dB insertion loss for the inverse-taper edge coupler;  $\leq 3$  dB insertion loss and  $\pm 2 \mu\text{m}$  alignment tolerance for the grating coupler.